

MAS125 Calculus II 2006-2007

Problem sheet for exercise class 9

- The instructor will present only one starred problem, leaving you more time to work on the other problems, which are similar to the ones in the assessed coursework.
- Since there is not time to do all the problems in the tutorial, it is important to study the solutions once they become available online at 3pm on Friday.
- Although tutorial problems are unassessed, they are in the same style as the problems in the final exam, so you are advised to study all of them very carefully.

(*)Problem 1: Find the absolute maxima and minima of the function $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ in the triangular region bounded by the lines $x = 0$, $y = 2$ and $y = 2x$. You should examine the behaviour of f at both the interior points and the boundary points of the triangle.

Problem 2: Find all the local maxima, local minima and saddle points of the following functions:

$$(a) \quad f(x, y) = 2xy - 5x^2 - 2y^2 + 4x + 4y - 4; \quad (b) \quad f(x, y, z) = 9x^3 + y^3/3 - 4xy.$$

Problem 3: Find a parametric equation for the line tangent to the intersection of the surfaces $x^2 + 2y + 2z = 4$ and $y = 1$ at $(1, 1, \frac{1}{2})$.

Problem 4: Calculate how much the function $f(x, y, z) = \ln \sqrt{x^2 + y^2 + z^2}$ will change if one moves a distance $ds = 0.1$ from $(3, 4, 12)$ in the direction $3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$.

Problem 5: Find the linearization $L(x, y)$ for the functions $f(x, y) = e^x \cos y$ at the point $P_0(0, 0)$ and hence find an upper bound for the error E in approximating $f(x, y)$ by $L(x, y)$ over the region $|x| \leq 0.1$, $|y| \leq 0.1$. (You may assume $e^x < 1.11$ in this region.)

Problem 6: Find the linearization $L(x, y, z)$ for the functions $f(x, y, z) = xz - 3yz + 2$ at the point $P_0(1, 1, 2)$ and hence find an upper bound for the error E in approximating $f(x, y, z)$ by $L(x, y, z)$ over the region $|x - 1| \leq 0.01$, $|y - 1| \leq 0.01$, $|z - 2| \leq 0.02$.

Problem 7: Find the maximum value $f(x, y) = 49 - x^2 - y^2$ on the line $x + 3y = 10$ by eliminating x in the expression for f . Show that one obtains the same result by using the Lagrange multiplier method.

Problem 8: Use the Lagrange multiplier method to find the maximum and minimum values of $x^2 + y^2$ subject to the constraint $x^2 - 2x + y^2 - 4y = 0$. (In this case, elimination of x or y would not be straightforward.)